A manpower planning model to predict future workforce behaviour and retention: A Markov chain approach

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ABSTRACT

Proper manpower planning is a key factor in the demand and supply of workforce in every organization. It probes the skills and capability sets, the right quantity, location, timing, and quality of the needed manpower. It is evident according to empirical studies that undivided attention has been given to manpower planning in the last decade. Different bodies involved include government parastatals, academic and industrial organizations and institutions performing some form of manpower planning research and activities to maintain stability and retention in the system. Whether these inputs are reflected in the manpower policies and make a significant contribution in this regard is yet to be seen.

Several analytics and methods of modelling manpower planning exist, however, Markov chain has been used widely and accepted in various facets and domains.

In this research, Markov Chain is used as a tool to analyse the manpower data from an academic institution as a case study with the aim to unearthing the hidden details regarding existing manpower policies and hence, its fairness and robustness towards staff training, promotion, and ultimately retention.

A 9-year stage of manpower data split into states is used and matrix operations employed in analysing the manpower data obtained. Bayesian probability is also used for establishing the transition probability matrix (TPrM), and these matrix transformations are carried out repeatedly to achieve stability.

The results of the analysis show that manpower policy in the participating organization towards overall staff retention is ***rigid and stern***. The results clearly satisfy the purpose of the study which is to predict the trend in the manpower practice, the potential cause of manpower loss and subsequently, the flow and fairness of the existing manpower policy.

***Keywords:* Fundamental matrix, manpower planning, Markov chain, absorbing, transient, Transition probability matrix, behaviour**

DEDICATION

This work is dedicated to God Almighty for His infinite mercies and unconditional love towards me and my family. I will forever be grateful.

Finally, this dissertation is dedicated to my beautiful wife, Chinenye Kate Okaekwu, and my gorgeous daughter, Chandel Muna Okaekwu. Your support and patience through the period is highly appreciated.

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ABBREVIATIONS

Abbreviation/Symbol	Description
CWDFC	Construction Workforce Development Forecasting Committee
DoD	Department of Defence
EE	Exponentiated Exponential
HR	Human Resources
MRT	Mass Rapid Transit
TPrM	Transition Probability Matrix

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CHAPTER ONE INTRODUCTION

One of the foundational aspects of engineering management is Organization which can be integrated or distributed. Engineering managers need to understand the nature and functions of an organization to determine management requirements and in managing resources as discussed in Okaekwu (2013). These resources could be financial, human, information, and physical resources. The "Function of Organization" points to having a full grasp of how businesses manage resources available to the benefit of the organization. One of these functions is the management and planning of manpower resources.

Efficient and proper planning of the manpower systems in the management of large and medium type organizations is a daunting task. Most of the organizations are either in the private or public sectors, for example, in engineering companies, and in the various University communities. There are different domains and segments in these organizations with different job roles and responsibilities. For instance, there are engineering roles in a non-engineering company, as in the case of a University having an Engineering Faculty and engineers as lecturers and as part of the workforce; also, there are engineering roles in a pure engineering organization where the majority of the workforce are engineers. Various skill sets are required at different levels to keep these systems running effectively. These skill sets can be brought with, when being recruited into the system, or can be developed through series of training, or from an extended period of service in the same system.

The Australian Government, Department of Defence (DoD) (2005) argued that given that a training system equates to a manpower system, it is of paramount importance to uphold the purpose of such workforce planning. This is to ensure that the right minds are retained at the training facility promptly to carry out the corporate strategy with most desired quality as discussed by Khoong (1996).

To cater for these needs without any form of shortfall or overflow of manpower, it is vital that the future expectations of the workforce can be forecasted to achieve the desired end. The output of this prediction should be reviewed into the existing manpower policy by management to promote retention. However, this is a tough task that requires some form of modelling to achieve.

1.1 Overview of Models

Some models focused on recruitment as a means of compensation for loss of manpower, which in turn, needs appropriate planning. Mythili and Ramanarayanan (2013) found, using numerical solutions, the recruitment time and the expected time to recruit. Parthasarathy and Chitra (2014) discussed that the main factors that determine the behavioral pattern of the workforce or manpower are *recruitment, promotion, and attrition*. They further expounded on the Exponentiated Exponential (EE) distribution with two parameters, (scale and shape parameter), focusing on the third order of the shape parameter. In their work, Parthasarathy et al. (2010) showed that for a single grade manpower system, the EE distribution can be used to determine when the total or aggregate loss of manpower goes above a random set threshold level, and it becomes counter-productive to the organization. Similar researchers as in Nancy et al. (2014), Parthasarathy and Vinoth (2009), and Mythili and Ramanarayanan (2012) have done extensive work in this area.

Different mathematical models have been introduced and used in the time past to model manpower. An overview of these models can be seen in but not limited to Price et al. (1980), Georgiu and Tsantas (2002), and Purkiss (1981). The Australian Government, Department of Defence (DoD) (2005) employed an exploratory approach in looking at different models for manpower planning in the military domain (Army specifically). The DoD is of the opinion that these models have their limitations but are well suited for the purpose in which they are used.

One of these models is simulation models. They are deterministic and are seen as a planning tool for an extended period, in tens of years and are very potent in the study and applications of complex systems as in Winston (2004). They are useful in various engineering, and military spheres. Winston argued that while they are excellent in answering the "what-if" questions and robust in handling complex systems difficult for analytical models, they are not cost-effective and are also time-consuming. Also, the method of the autocorrelation of the output data is hard to apply.

Besides, in optimization models, there exist different optimization techniques which include dynamic, linear, integer and goal programming skills. They are used in the design of policies of a time-dependent sequence of decisions or a one or multiple-objective decision issues as discussed by Winston (2004) and Budnick (1988). The setbacks in the techniques are found in the fact that real world manpower planning situations have *several desired objectives* as stated by DoD (2005). And in practice, high skills are needed in the implementation and translation of a decision problem into mathematical programming formulation as noted by Budnick (1988).

Even though Markov chain has its drawbacks because of lack of internal mathematical programming techniques to optimize outcomes as stated by DoD (2005), as a state variable, they are more suitable for the prediction of the future behaviour of manpower systems in which staff movements are not specifically controlled. This is the main concern of this study. The descriptive or exploratory modelling process involves the description of stock and flow process from one state to another following the establishment of those states. For example, one may wish to describe how a transfer or retirement process works. However, when we look at a proportion of staff members in each state, it then leads to a fractional flow model; which in turn becomes the Markov model if we take the proportion as a probabilistic function as noted by Price et al. (1980). Other models will be discussed further in the review of literature in order to establish the suitability of the Markov model in the context of the study.

1.2 Problem Statement

A *practical problem in an engineering organization* is the manpower loss of staff members. The cause of the losses is sometimes unknown. It could be due to unfavourable manpower policies that might exist in the organization. This underlying cause can be deciphered through the analysis of the existing manpower data in the organization. However, the selection of appropriate analytics needed for unearthing the underlying trends hidden in manpower planning data is intellectually challenging.

The selection of appropriate analytics needed for unearthing the underlying trends hidden in manpower planning data is intellectually challenging. Markov chain model conceptualizes the movement of staff in an organization, over the course of the history of transition, as a ***stock-and-flow*** progression. It employs matrix operations to analyse manpower data obtained. Markov is a state variable and as such offers a better approach towards discerning trends in manpower flow. Thus, Markov chain is a kind of transform that achieves data dispersion to gain insight into the prevailing nature of manpower phenomenon.

Proper application of this model enables us to forecast the direction in which manpower practice of an organization is tending towards, for example, how much manpower flows to retirement and wastage respectively as absorbing states.

1.3 Purpose of Study

The goal of this study is to predict the trend in the manpower practice, the potential cause of manpower loss and subsequently, the fairness of manpower practice or policy in the organization under study. To achieve this purpose,

(i) Nine years (stages) of manpower data (see Appendix E) split into a number of states is obtained and namely:

- Employment;
- Staff strength/Total Stock;
- Training;
- Promotion;
- Resignation;
- Retirement; and
- Termination.

The data was extracted from the primary records of the participating organization. There could be other states as we may wish to establish. The first four represent the *non-absorbing* (transient) states while the last three represent the *absorbing* states.

(ii) Bayesian probability was used to establish the transition probability matrix (TPrM). The Bayes rule is arguably the most famous rule of the probability theory. Its popularity comes from the fact that it provides the important process of human reasoning which is essentially "learning from experience". It can tell us how we should modify the strength of our belief in a particular hypothesis after we have learned some new evidence. The Bayes rule follows the general rules of conditional probability. According to Tijms (2004), Bayesian statistics finds application, from its use by accountants and tax inspectors in their audit duties. The Bayesian probability method is used to calculate the different probabilities of transition from one state to the other in this work.

(iii) Matrix transformations, which are carried out repeatedly to achieve stability in the transition probability estimates, was used. To achieve stability, the class or sample size must be fairly large as stated by Heneman and Sandver (1977). Equations 1-3 applies as in Page (2006) and Kemeny and Snell (1976).

Then, F = $(U - Q)^{-1}$ .... (1)

And G = FR .... (2)

Where T = $\begin{bmatrix} \frac{(U|O)}{(R|Q)} \end{bmatrix}$ .... (3)

U = Unity matrix;

O= null matrix;

R, Q = Random matrices;

G = Solution matrix; and

T= TP_rM (Transition Probability Matrix).

The research described in this work, combined with findings from related literature, will support current work done by past researchers.

1.4 Research Objectives

The use of Markov chain model as the appropriate and selected analytics for extracting the underlying trends hidden in the applicable manpower planning data is supported by relevant literature as discussed in the preceding sections. Thus, it is proposed as the right model for determining an organization's staff transitions from one state to another as reflected in the research questions.

Therefore, the overall objective of this research is to use this model to answer the research questions in predicting:

- Staff retention level;
- The presence of structures critical to staff's job satisfaction; and
- The effect of organizational policy towards overall staff development.

1.5 Research Questions

The research is quantitative in nature and thus, require hypothesis testing. It aims to answer the following fundamental questions:

- Question 1

How much manpower flows to retirement and wastage respectively as absorbing states?

- Question 2

  How often do members of staff get trained and promoted?

- Question 3

  Does the lack of training and promotion lead to any form of attrition?

1.6 Research Scope and Limitations

The study sought to establish a view on the theoretical foundation for a better understanding of manpower planning, its importance in an organization, and the application of Markov chain in analysing its manpower data.

The research then looked into details to a particular organization, in this case, a South African University with the aim of establishing the purpose of the study as stipulated in section 1.3 above. This entailed the use of a case study approach. The considered organization is a well-known one in the education/academic sector, and the researcher is also very well acquainted with the organization in question.

One major limitation of this research was the difficulty faced during the data gathering phase. The participant organizations were very reluctant with releasing the required data as it relates to human resources. This was quelled after the ethics clearance and consent letter was sent through.

Furthermore, the researcher would have loved to have data that spans through to say between 10 - 20 years, however, the researcher is limited to a 9-year data because the organization cannot trust the data before the initial date used in this study. The participant organization claimed that they only started keeping accurate data when they instituted the current technology being used in the organization.

1.7 Area of Study

It is envisaged that the research will depend on data obtained from a South African organization (University) for ascertaining the goodness of fit of the model to the data.

1.8 Significance of the Study

Du Toit (2006) stated that South African is gearing towards a service-based economy from a resource-based one. However, proper resource planning is key to enhanced services, and therefore, the development and retention of personnel in any organization become very crucial. As stated by Du Toit (2006), there is an urgent need to develop people in the South African context. Therefore, an effective manpower planning becomes an imperative for any South African organization.

It will be of great benefit to note the dynamics of the flow towards attrition in an organization; thereby giving a clear direction towards effective manpower planning within organizations. Organizations gain competitive advantage through effective utilization of manpower recruited from the labor market. Good manpower practices help to attract and retain a workforce for achieving corporate goals.

This research, therefore, seeks to ascertain where and how effectively a designated South African organization stands on its manpower planning. The output of this research will be insightful especially to other South African organizations in the same domain.

The designated organization has been around for many decades and has operated in the academic sector in South Africa successfully with thousands of employees. The success and value in this industry lie mainly in its workforce.

Irrespective of the outcome of the research, it intends to crystallize the need for proper manpower planning in South African organizations by displaying how the designated organization goes about its business in that regard.

1.9 Definition of Terms: Terminology, Notation, and Symbols

1.9.1 Terminology

State: Job status or position subject to transition
Stage: used to mean one year at a time
Wastage: termination, sack or dismissal without severance packages
Retirement: planned and complete stoppage of employment with benefits
Absorbing State: Probability of return is zero - point or (state) of no return
Non-Absorbing State: State of transition or movement from one state to another

1.9.2 Notation

v : A unique unit vector; **also eigenvector**

μ, μ_v : column vectors

F: fundamental matrix

F_v: variance of fundamental matrix

F_{sq} : square matrix (every element of the matrix is squared)

F_{dg}: diagonal matrix

μ: is the estimated number of transitions among non − absorbing states

μ_v : variance of transition among non − absorbing states

O: zero matrix

U: A unit matrix

R: Random matrix signifying movements from transient (non − absorbing) to absorbing states

Q: Random matrix showing movements from transient − to − transient states.

All notation is as in Kemeny and Snell (1976).

1.9.3 Symbols

$\sum_{j}^{k}$: summation of states from j to k.

1.10 Conclusions and introduction to next chapter

Having done a brief historical background work and looked at the overview of models, we have also established the significance of the research and its relevance in the South African organization. In the scope of the study, we pointed out that a designated organization is thoroughly examined through our clearly outlined objectives. Research limitations, problem statement, purpose of study and research methodology are also presented.

However, before we can achieve any meaningful conclusion, there needs to be a full grasp of involved concepts. This is what we hope to accomplish in the next chapter where the research kicks off with reviewing all pertinent literature relevant to the subject matter.

CHAPTER TWO LITERATURE REVIEW

The chapter gives a fundamental review of the applicable literature related to Markov chain and manpower planning. The historical review and development of general manpower planning along with its background and that of stochastic processes are presented. A detailed review of models and discussion of applicable literature are mentioned. Definitions of key terms establish a standard terminology for the remainder of the technical discussion.

2.1 Background and Historical Review

There was a growing awareness in the early sixties, followed by a significant growth from mid-sixties to the seventies in the area of manpower planning. The techniques to manpower planning hinge on several assumptions which point to but are not limited to human behaviour, economic influence, and decision making for management. A detailed historical review of manpower planning, of the United Kingdom specifically can be seen in Smith and Bartholomew (1988).

Figure 1: Basic Manpower System - Parmar and Makwana (2012)

Bartholomew (1971) noted that "*manpower planning is an interdisciplinary activity.*" For this reason, it requires the technical know-how of subject matter experts in different fields with the aim of making predictions of the demand and supply for individuals with these skill sets and qualifications and bringing them to productive use. Not only that, but also to look at the manpower policies to resolve the discrepancies between the demand and supply of manpower commonly known as "*closing the manpower gap*" as described by Edwards (1983). Parmar and Makwana (2012) described a typical manpower system and a realistic system as shown in figures 1 and 2 respectively:

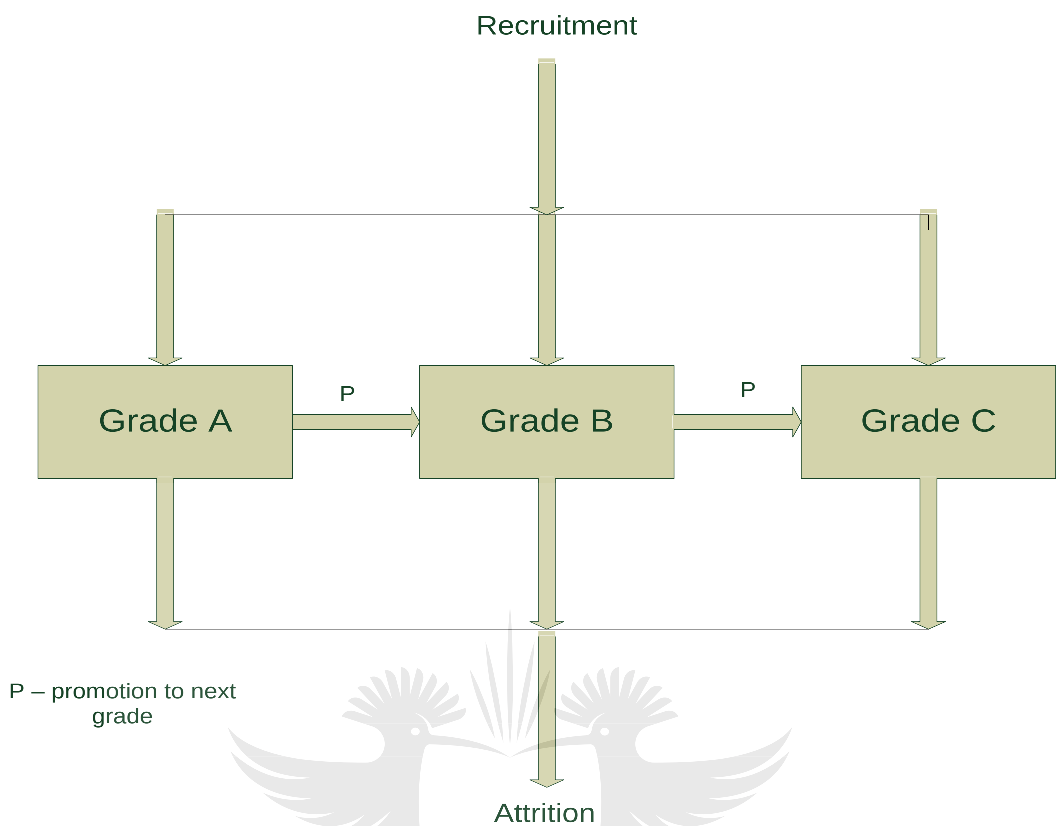

Figure 2: A realistic model - Parmar and Makwana (2012)

Figure 1 shows a direct relationship between the stock and the flows of manpower system whereas figure 2 shows a more elaborate concept of the movement of manpower through different stages after being recruited towards wastage.

2.1.1 Background of Stochastic Processes

According to Lawler (2006), a stochastic process is a collection of variables whose predictions cannot be done with certainty. Specifically, as stated by Pinsky and Karlin (2011), it is a random process that makes use of random variables X_t; where t is a parameter indexed by time T, given the evolution of time. Moreover, stochastic processes are differentiated by their immaculate state space or by the possible value range for the random variables X_t indexed by time set T, and also by the interdependence amidst the random variables. This process takes the form of discrete time, time homogeneity, and Poisson processes which share the same property with the Markov process.

2.2 Definition of Key Terms

It is of paramount importance that one should give a definition of specific terms at the commencement of a discussion carefully, so stated Gordon H. Clark, the American philosopher. This restriction as Clark (2002:138) noted stems from his belief that one does not fully know about a subject matter unless one knows the definition. This also applies to the realistic world of engineering management. Various definitions and theorems regarding Markov chains have been given in Kemeny and Snell (1976). We will give in this work, a general definition of these subjects for easy understanding.

2.2.1 Absorbing State

An absorbing state is a point of no return. It is a state in which upon entrance, there is no chance of transition. Therefore, a state ***z,*** of a Markov chain is absorbing if $P_r ii = 1$.

2.2.2 Fundamental matrix

For an absorbing Markov chain, the fundamental matrix F, given by $F = (U-Q)^{-1}$, is the anticipated number of times the Markov objects make a visit to different states before going into absorption Kemeny and Snell (1976).

2.2.3 Transition Probability Matrix

As described by Pinsky and Karlin (2011), the one-step transition probability matrix and the probability distribution specified on the state space at time ***0*** defines the Markov chain. The Markov chain analysis is concerned mainly with the computation of the transition probabilities and is central to the calculations; and therefore, the transition probability matrix equation is as shown below in (5). Pinsky and Karlin (2011) further discussed on relevant theorems. Equations 4 – 6 is given by Pinsky and Karlin (2011).

If we consider a Markov process $\{X_n\}$ on a finite state space ***N*** and the fundamental properties of the conditional probability,

$P_r\,[X_{n+1} = j \mid X_n = i_n]$ ………………………………………………………………………………. (4)

Equation 4 gives the conditional probability $P_{ij}(n, n+1)$ as the one-step transition probability from state ***i*** to ***j*** at time ***n*** as given by Kijima (1997); and the one-step transition probability matrix is given by

$P_r\,(n, n+1) = (P_{ij}(n, n+1))$ ……………………………………………………………………. (5)

As stated by Kijima (1997), from (2) and (8), the right-hand equation $(P_{rij}(n, n+1))$ is equal to

$(P_{rij}(n, n+1)) = P_r[X_{n+1} = j \mid X_0,\ldots,X_{n-1}, X_n = i]$ ……………………………………………………….(6)

Equation 6 is a crucial equation for us in determining the transition probabilities in this study. For *m-step* transition probability at time n, we will simply replace the number ***1*** in (5) by ***m*** for the corresponding m-step transition matrix. A complete mathematical derivation can be seen in Kijima (1997). Similar explanations were given by Pinsky and Karlin (2011). They also gave the condition for stationarity of the transition probabilities as when the one-step transition probabilities are not dependent on the time variable ***n***.

2.2.4 Absorbing Markov Chains

Following the definition of the absorbing state above, a Markov chain is said to be an absorbing one when it meets the requisite conditions as indicated below as indicated by Pinsky and Karlin (2011):

- **Condition 1**: that the chain has one or more absorbing state; and
- **Condition 2**: that there is a possibility of transiting from one non-absorbing state to an absorbing one; with also the possibility of doing that in more than one step.

2.2.4.1 Properties of the absorbing Markov chains

The absorbing Markov chain has the following under-listed properties Pinsky and Karlin (2011):

- Given a finite number of transition steps, the chain will transit into an absorbing state irrespective of the initial state;
- The initial state of the chain gives a long-term trend of the chain;
- Like the regular Markov chains, absorbing Markov chains possess the property that the powers of the transition matrix tend towards a limiting matrix; and
- The product of the fundamental matrix F, and some random matrix concerned with the transition from transient (non-absorbing) to absorbing states ***R***, gives the solution matrix, **G = FR**.

2.2.4.2 Transition Matrix for an Absorbing Markov Chain

The transition matrix of the absorbing Markov chain is a square matrix depicting the probabilities of transition from one state to another in a system with, at least, one absorbing state as described by Kemeny and Snell (1976).

Having discussed the absorbing Markov chain which forms the core of our analysis, we present the standard form of the transition matrix of the absorbing Markov chain. The absorbing Markov chain is a standard form if the labeled rows and columns are such that all the absorbing states are followed by the non-absorbing ones, partitioned into four sub-matrices as shown below:

$$T = \left[\frac{(U|O)}{(R|Q)}\right] \qquad (7)$$

where the parameters have been defined in section 1.3 and the equation is culled from Kemeny and Snell (1976).

2.3 Probability theory

Probability can be defined as the equiprobable favourable over a total number of possible outcomes.

Tijms (2004:229) noted that before the Russian Mathematician (Andrej Kolmogorov) laying an acceptable mathematical establishment of the probability theory, the mathematical constructions of the basis had, over a long period been a trial and error process. Kolgomorov, according to Tijms (2004) formed the axioms which make the foundation for the mathematical probability theory.

One of the fundamental theoretical concepts in probability theory is the *sample space* whose subset is an *event*. A numerical probability is assigned to each event to constitute a probability measure of a function ***P*** as noted by Tijms (2004:28). This probability measures and applicable probability theorems have been discussed thoroughly by Kemeny and Snell (1976, 6:13), and this forms the basis of its application in this study.

2.4 Manpower Planning in Engineering Management

Manpower planning is a broad term that has been used over the years in the field of human resources to represent the careful forecasting of the required number of personnel, with the right skills, in the right job position, at the minimum cost and at the right place and time. As discussed by Halim and Zeki (2012), this is driven by different factors like changing business needs, technological changes, and the make-up of the workforce planners in various organizations.

The management of organizational manpower must be informed by both its internal and external dynamics. Various literatures have treated the issues of manpower planning extensively. The most occurring issue is the planning and control of manpower grade sizes which has to do with the provision of high quality and quantity of manpower required to meet organizational objectives as stated by Price et al. (1980). As discussed by Guerry (2011), manpower planning is an aspect of manpower management achieved through some mathematical modelling. He further explained the modelling regarding three flow types which include recruitment, internal personnel movements, and wastage. Santos et al. (2009) presented a system of workforce planning for HP's IT business that focused on supporting labor demand and supply matching while putting costs at the barest minimum.

The alignment of manpower policy with business objectives is achieved viz-a-viz a response from the manpower system and reconsideration of organizational objectives. The implication is that manpower planning needs to function in various circumstances that cannot be mostly predictable. Therefore, it is imperative that manpower management seeks the right tools or models to cater for these needs and come up with alternative strategic decisions according to Purkiss (1981). These models can be exploratory or normative which ranges from the very simple to use to very powerful ones. They help to point manpower managers to the right direction, giving a full understanding of how the workforce works and different circumstantial responses. They are also powerful in the sense of their use in stochastic simulation and the computation of a set of manpower data as will be seen later.

According to Flores et al. (2007), better business strategies can be achieved by effective manpower forecasting techniques. This makes manpower forecasting, an essential aspect and practice for various firms and organizations including government parastatals. Different techniques for manpower modelling and forecasting exist, based on the needs of manpower according to Park et al. (2008).

Cipriano et al. (2009) argued that proper manpower planning and scheduling is founded on forecasting supply availability and job requirements to deploy the right workforce, with the right skills set, at the right time and given the right place and cost.

Siti (2012) noted that workforce planning is the combined effort of planners and line management; which brings with it, increased responsibilities and challenges as they try to develop programs to serve the needs of the business and give direction; as well as short and long-term solutions to manpower concerns.

The traditional style in manpower planning within organizations focuses on the right people, right position, right time approach. However, due to other factors like economic demands, demographic movements, environmental and natural effects, this style pales in comparison to using the right technique to enable management in making strategic decisions, generation of alternative manpower policies; and ultimately, aligning employment policy towards the achievement of set strategic goals.

The traditional style is myopic and lacks the much-needed power to inform. Therefore, we employ an approach in generating outcomes that will help in the formulation of a normative model that point to the desired end.

2.5 General Planning in the University Context

The University is set up primarily as a training system; therefore, it is important that a proper workforce be assembled to achieve this primary aim.

The Great Britain, University of Sussex (1972), in the article, "Planning in a University", Geoffrey Lockwood, the planning officer, argued that *planning is partly forced upon universities*. He further assumed that the absence of the planning techniques required means absence of planning since planning is most times associated with techniques. He also argued that Universities, more than other institutions, have a pressing need to plan. This is because, the lack of it, haunts the institution decades later. He pointed that each member of staff should be given the opportunity to contribute their quota towards the planning of their domain, hence, the need for proper manpower planning. This, he classified under "contents" in his description of the planning process.

In their article that looks to provide a framework for strategic manpower planning in the academic domain, Ulferts et al. (2009) stated that modern manpower planning is concerned with forecasting of the organization's manpower needs for the future and establishment of its objectives. It also requires the development and implementation of specific events like recruitment, promotion, and training. This is to make sure that the right people with the right portfolios are available as and when required. One aspect of the strategic manpower planning process is forecasting manpower requirements for the future centred on the strategic goals of the institution. As stated by Ulferts et al. (2009) below, vital questions to be asked include but not limited to:

- *"How many faculty and staff will be required to achieve the strategic goals of the college?*
- *What jobs will need to be filled?*
- *What skill sets and credentials will people need?*
- *What motivates the staff members the most?"*

Manpower modelling and forecasting tend to answer these questions and, in turn, provides the management personnel the vital information necessary regarding the implications of various manpower strategies that can be employed to reach institutional goals as discussed by Bechet and Maki (2002). Another area looked at as part of the process of the manpower planning is in the "*closing the talent gap*" as described by Brush and Rose (2005). To achieve this, organizations need to look at the situation as the decision of building: which requires training and promotion of internal resources as opposed to buying: when the need for talent is urgent. Kesler (2000) echoed the same sentiment in sticking to an approach that connects and matches business needs with the resource requirements, drives organizational inputs towards objectives that give the optimal output, provides a framework for manpower resource function across all business units in the institution, to mention but a few.

To understand the overall manpower supply planning, it is also important to discuss some previous research focused on manpower resource scheduling. In the past, several research has been done on resource scheduling especially on literature in the area of airline as seen in Langerman and Ehlers (1997), Yan and Tu (2002), and Yan et al. (2002, 2004, 2006a, 2006b, 2008a). In mass transit as seen in Yan et al. (2006c and 2008b) and Higgins (1998); and generic scheduling according to Aykin (2000), and Alfares (1998).

Ekhosuehi et al. (2015) looked at recruitment distribution about the different standard grading as stipulated by a Universities' Commission in Nigeria. This, they did by circumventing the possibility of what they called "negative recruitment" (wastage and redundancy). In their study, a close look at the Department/Faculty under study shows a high probability of being stagnant in one position, that is, at the minimum, a stagnation from one category to the next higher category. While an attempt had been made to generate the desired structure, not considering negative recruitment, which is an important factor, one would be more concerned about how this structure can be maintained and preserved.

2.6 Discussion and application of theory

2.6.1 The Markov Model

The Markov model looks at how the world has evolved over time. It treats the world as a collection of system of states and considers transitions between these states, given some statistics about how these state transitions are set up. The Markov model looks at the given statistics, that is, the uncertainty or the probability of transitions and looks at how the system in question evolves over time. This points to how much time is spent in different states as discussed by Pinsky and Karlin (2011).

Edwards (1983) listed four features of a good model, and the Markov model meets these requirements. Further theory regarding the Markov chain is seen in Norris (1997:128-159) with the treatment of the subject matter covering areas like the Martingales, Brownian motion. Further treatment is found in the field of electrical networks, having a countable set of nodes, with each node having a capacity greater than zero; and in the Potential theory which extends into the fields of electrostatics, the flow of fluid, and heat diffusion.

Pinsky and Karlin (2011:87) stated that in the past, significant instances of biological, economic and natural physical phenomena have been modelled by Markov chains. Several models based on the Markov chain model were also discussed. He highlighted the inventory model which finds application in a situation where commodities are stocked for continuous demand satisfaction. Also, as noted by Pinsky and Karlin (2011:89), the Ehrenfest model is also a type of Markov model which is a classical description of diffusion through a membrane.

The prediction of supply or withdrawal of the workforce in an organization can be made using Markov chain to model the flow of personnel through specified levels which can be based on skills, status, and the likes according to Zanakis and Maret (1980). Such applications regarding the movement of staff are also found in Nielson and Young (1973). Merck (1970) also discussed manpower flow and especially based on the number of years the employee has served. Zanakis and Maret (1981), discussing manpower flow, combined the Markov process with mathematical programming for a more realistic model that can be used in the area of management science and operational research. This type of combined processes can also be found in Charnes et al. (1976:111-134). Several researchers have also adopted this combination as in Clough et al. (1971:299-315) and Purkiss (1971). Gallisch (1989), in his application, considered the civil servants in the defense administration of the Federal Republic of Germany. In it, he provided a solution that would predict the number of manpower with a consideration of the present availability and the probability of transitions between different states. The work of Raghavendra (1991) in his bivariate model for Markov manpower planning systems is found very useful. He highlighted policy dichotomy on promotions which is based on length of service and performance ratings. Markov applications are further seen in the case study of the Slovenian Armed Forces in Skulj et al. (2008), identifying 120 types military segments after which irregularities and disparities were seen in the projected arrangements of the military subdivisions and the official set targets.

2.6.2 Markov Process

A Markov process is a system that assumes the Markov property. It specifically states that the future or the next iteration can be predicted solely on what is currently happening. That is to say that we do not want to know what has occurred in the past, we only need to know what is going on right now to predict the future. This is the assumption. However, some Markov processes do consider the past to make a future forecast. In mathematical terms, we say that the distribution of X_{n+1} depends only on the current state $X_n = i_n$. Therefore, the process $\{X_n\}$ is a Markov process, according to Kijima (1997) if, for each *n* and every $i_0,\ldots,i_n$ and *j* and element of set N, then,

$P_r [X_{n+1} = j \mid X_0 = i_0,\ldots, X_n = i_n] = P_r [X_{n+1} = j \mid X_n = i_n]$ ……………………………….……… (8)

Equation 8 is also according to Kijima (1997).

2.6.2.1 Discrete-Time Markov Chains

A Markov chain is a way of modeling a system. The main property of Markov chains is that it has states and transitions. The transition is given by the probability of transitioning between two states. Markov chains are of different times. These times can be discrete or continuous. Markov chains are better described through simple illustrative diagrams. Norris (1997) describes Markov chains as "*the simplest mathematical models for random phenomena evolving in time.*" He defined the discrete Markov chains and investigated their behavior with various theorems bordering on the Markov property and their inherent characteristics. Similar theorems were also given by Kijima (1997) to show transition probabilities using the strong Markov property and homogeneity to establish a relationship within unknown variables. The Markov chain described here is of discrete-time, defined on a finite state space N = {0, 1, 2…., N} given that N is less than or equal to infinity; and on the assumptions that:

- the employed Markov chain is homogeneous; and
- the state space, N comprises non-negative integers.

2.6.3 Application of the Markov Chain model

Since the first application of the principles of Markov chains by Seal (1945) towards the depiction of a manpower configuration behavior, tremendous development, and utilization of these models have abetted in the popular and efficient planning and control of manpower attributed activities. This has, in turn, led to an improved capability set in an organization or system to meet the organizational objectives as pointed out by Dimitriou and Tsantas (2010). In these models, depending on the different features and traits of the population of the organization as described by Ugwuowo and McClean (2010) and Feyter (2006), classes or states are used to depict the fragmentation of the population into several homogeneous groups. In various organizations as in the case of Universities or the Army, for example, where the existing workforce can fill most positions, then there exists a hierarchical model in which different grades are used to form these classes as seen in Feyter (2007).

Markov chain has found applications in various fields. For example, in a new rapid transit system, the system is analysed to see the usage of the new regime compared to people using automobiles. In sociology, the rapid transit system is used to analyse the classification of people by income as exemplified with analytical data in Glass and Hall (1954) and further analysed in Kemeny and Snell (1976). Markov chain analysis is also used, in the absence of clinical study, to effectively carry out an evaluation of involved risks and benefits of different strategies used in medical treatment as discussed in Sox et al. (1988). This is especially seen in physics where physicians are faced with life-threatening decisions to make in an "*either/or*" situations after diagnosis. It has also been used in the modeling of student retention at colleges and universities.

Norris (2007:170-206) dealt thoroughly with the applications of Markov chains in different spheres of life. He pointed out its application in Biology, specifically in the Branching process whose own model has many applicable solutions to population growth issues. And also, in the chains reactions in chemistry and nuclear fission study. Still in the biological domain, he highlighted the Markov chain application in epidemics sighting that the "*decline of an epidemic can also be explained by the eventual decline in the number of individuals susceptible to infection, as infectives either die or recover and are then resistant to further infection*."

He further sighted in the biological sphere with Wright-Fischer model which employed the discrete-time type of Markov chain on each generation given a finite set, and also in the Moran model of birth-and-death chain.

Chemistry is another domain where Markov chain has found its application quite useful as in the classical model of enzyme activity. Kutchukian et al. (2009), in their work, developed an algorithm to transition the fragment of a molecule undergrowth to a new state which is unaware of its past based on Markov Chain. Similarly, the work by Kopp et al. (2012) suggested that Markov chain can be used to model accurately the growth of some oxide materials having a periodic structure of layers of two or more materials.

Pinsky and Karlin (2011:90) discussed its application in genetics in conjunction with the genetics model introduced by S. Wright to investigate the fluctuation of gene frequency under the influence of mutation and selection.

Further application as discussed by Norris (1997) is found in queues and queueing networks whose mathematical model is based on arrival rate and where priority is given in first-in-first-out order. For example, in a telecommunications system, when there is traffic of calls from customers, according to Meyn (2007), the calls are queued in the order of arrival, and clients wait until a resource is available and priority is given to earlier arrivals. The assumption is that the times between arrivals and the times taken to take customers' calls are independent random variables and can be modelled using the Markov chain.

Markov chain finds application also in resource management. Examples can be seen in restocking a warehouse, reservoir model, and in the planning of manpower which is the main focus of the study as described by Norris (1997).

2.7 Discussion and application of applicable literature

2.7.1 Further Review of modelling theory

Other models like the normative models take the very next step forward in forecasting requirements and informing the management decision in order to achieve set target objectives. The most being the linear programming where basic equations required to maintain stock levels must satisfy the equation of the type:

$x_3(t + 1) = x_3(t) + P_{23}(t + 1) - A_3(t)$ .... (9)

$x_2(t + 1) = x_2(t) - P_{23}(t + 1) + f_{12}(t + 1) - A_2(t)$ .... (10)

$x_1(t + I) = x_1(t) - P_{12}(t + 1) + r(t) - A_1(t)$ .... (11)

Where x_1, x_2, x_3 are respectively the desired fixed number in each state and for each period. R(t) represents recruitment of new staff which we assume happens at the lowest level, and promotions P between levels are depicted by P_{12} and P_{23} respectively. We can estimate attrition denoted as $A_1(t)$, $A_2(t)$, and $A_3(t)$ respectively. In the equations (9), (10), (11) in Price et al. (1980), above, all of the parameters are fixed and known except for the P-variable. The later generation of the normative model uses network algorithm codes which have speed and graphics for an explanation as the main advantage.

According to Price et al. (1980), renewal-type models are used for examining policies relating to growth rates and evaluation of their application results on specific parameters like rate of promotion, and how long one has spent in each grade. This model is best suited for when recruitment and promotion are internally controlled and decisions made specifically for filling available roles in the organization. It must also be noted that in situations where high costs are prevalent and need to be resolved, then the best models will be goal programming models (optimization model).

Cappelli (2007) studied manpower planning from a supply chain approach. In the 60's and 70's, manpower studies stemmed from the requirement of businesses to produce reports on the expected resources to avoid shortfalls in the level of skilled staff that could be a setback to production. He was of the assumption that the supply of manpower in an organization is a dedicated and an internal function in those days. The planning stems from a forecast of future supply matched against the demand of manpower internally which is a measure of the growth of the organization. It also points to internal progress and shows the rate of staff transition from one job position to another. However, there was a decline in the planning of manpower in the following decade when the ability to forecast was battered due to the oil surprises in the mid 70's. Due to the recession that followed, there were so many talent cuts which meant that there was a surplus supply of talents when organizations needed to re-hire. This means that there was no need to plan as availability was confirmed at the organization's beck and call. This was evident in the slashing of manpower planning and talent management functions. However, few organizations maintained a low level of workforce planning using different models like statistical regression model, Markov chain model, and operational research tools for talent needs forecast. Park et al. (2008) developed an integrated manpower forecasting model which combines demand and supply of manpower using a System Dynamics methodology; and incorporating a feedback structure, time lags, and a flexible saturation point for the information security industry. The system dynamics model helps to understand the problems with the dynamic behaviour and to investigate various system dynamics, focusing on the feedback structure of the scheme. The simulation results reflect a shortage of manpower in the Information Security industry. Upon several iterations, the model showed alternatives towards achieving manpower stability between demand and supply. Gupta and Ghosal (2013) developed a probabilistic manpower planning model under varying class sizes focusing on the length of service based on promotion as compared to age. They argued that since the main goal of an employee in an organization is a promotion to the next grade, then it is better to consider the length of service rather than age as the criteria for promotion. Due to withdrawals or wastage, vacancies arise, and promotion is imminent. However, they also considered the size of the different grades which is mostly fixed given the amount of work at each class.

Lee et al. (2000) used an optimal control model towards solving manpower planning problem. The model aims to find the optimal way towards task scheduling, workforce wastage, and training assignments while maintaining the satisfaction of the minimum workforce demand. They provided the model in a mathematical and as a discrete-valued control problem which can be solved by the discrete-time optimal control approach. This novel mathematical formulation was considered a first attempt towards a large scale optimization model for real manpower planning problems. Mouza (2010) also did an empirical study towards the application of optimal control technique which she considered being the most efficient process for multi-objective programming. In this, she presented a solution method based on the generalized inverse *Lazaridis*. This solution, they believe would provide management and policy makers the necessary direction in determining the optimal path for resource reallocation, and strategy reforms.

Park et al. (2006) introduced a system dynamics model of manpower planning to predict the demand and supply of labour in Korea's information security industry. He identified that the critical factor in manpower planning is the ability to meet up with future demands and supplies of manpower. In the system dynamics approach which he took, he considered the feedback structure between demand and supply of manpower as he argued that previous researchers have treated the subject in an open loop manner. He termed this feedback mechanism as the "*supply-demand gap*". He also pointed out that time delay which can increase instability in the system as also pointed out by Sterman (2001) is another important factor to consider during the design of the manpower supply-demand model. In his simulation model, he classified manpower into grades based on the level of education and work experience with workers of each class designed to step to the next available grade after a given period of service in the organization.

Bhatnagar et al. (2007) presented a framework for optimal allocation of permanent and contingent workers towards different sub-processes through the use of a linear programming model. Other literatures regarding the contingent planning of manpower are discussed in Silva et al. (2000). In their aggregate production planning model, they pointed out that the frequency of recruitment and wastage are contributing factors to frustration among the workforce and has an undesirable impact on productivity. Other extensive reviews on optimal control and heuristic models can be seen in Anderson (2001), and Slomp and Mollleman (2002) among others. Also, Chen et al. (2010), in their paper titled "*Short-term manpower planning for MRT carriage maintenance under mixed deterministic and stochastic demands*", developed a two manpower supply planning model and a solution algorithm with a blend of both deterministic and stochastic formulation. This model was driven by the need to allocate their manpower resources more efficiently to cater for the mass rapid transit system under consideration. The planning process for the short-term maintenance resource supply is in three stages with the second being the manpower supply planning, and the most complicated as it is a precursor to the final stage.

Cappelli (2007) noted one important point which is the expected uncertainty of attrition which kicks in when nothing new is done about the existing workforce. The rate of attrition is uncertain and influenced largely by circumstances beyond the organization's control. This could be chiefly due to the plundering of competent staff by competitors. In recent times, the challenge for organizations is to determine what their internal supply would be, and in measuring their competencies to meet business needs. He also noted that one of the most difficult things to do in manpower planning is to forecast talent demand. He, therefore, proposed borrowing the techniques of the supply chain as a means of manpower planning having a similar task. Gresh et al. (2007) also implemented the supply chain optimization approach as a solution to manpower planning problems.

2.8 Discussion of literature findings and benchmarks

The perceived concept of the Markov model has been thoroughly discussed in the various pertinent literatures by renowned authors in Igboanugo and Onifade (2011), and Purkiss (1981) in this field. Woodward (1983), in discussing Markov manpower planning system based on grade, age, and service length class definition, provided a solution from several iterations to predict the future expectation of the required manpower for each analysed class. Igboanugo (2013), in his work, analysed the manpower data of a local government hub in Nigeria and found that it possesses the Markov property with stochastic regularity. This property has been discussed in the earlier part of the literature review. In trying to unmask the dynamics of the staff stock and flow in a typical Nigerian University settings, Igboanugo and Onifade (2011) analysed a forty-year manpower data and established the current manpower policy is "*liberal and firm*" and geared towards capacity building. Their work is geared towards uncovering the existing manpower policies and future directions. The work done in this regard is similar to the work by Igboanugo and Onifade (2011). With the application of the Markov model, which enables us to forecast the direction in which manpower practice of the University organization under consideration is tending towards, we shall provide a solution to predict the flow, and hence the fairness of manpower practice or policy.

One of the premises on which Belhaj and Tkiouat (2013) has based their model on is that the grades to which an employee can be promoted is limited and his entire career can be known since the career of an individual in an organization is related to one's level of study and qualifications. This is not true for all organizations. One of the reasons why this is not true stems from a political angle. Staff retention, promotion, wastage, and likes, as transient states, more often than not, have a political undertone. Their assumption that promotion happens to the next first higher grade does not apply to the organization under study. For example, a staff member who was hired with bachelor's degree can after few years obtain a doctorate and gain promotion to a level higher than one's counterpart with say a Masters degree. Against this backdrop, we have taken a holistic view of staff transition using Markov model for prediction. Belhaj and Tkiouat (2013) employed a predictive model, motivated by the reality of staff development in a hierarchical time-dependent manpower system; dividing the system into sub-groups using the popular Markov transition matrix. Belhaj and Tkiouat (2013) further argue that organizations must consider both internal and external supply of qualified candidates in forecasting the supply of its manpower resources. This is influenced by the manpower states under study which include but are not limited to retirement, promotion, mobility policies, training and manpower development. They addressed manpower planning by grouping the system into families of same grades so that there could be an acceptance that each resource of the same group will evolve in like manner. They further assert that promotion is key in resource retention, assuming that promotion is done only to the first next higher grade.

Wong et al. (2012) in their paper, reviewed critically various forecasting models used for predicting manpower demands by looking at their strengths and weaknesses, and rationale. The review was focused on the construction industry and training was identified as a driver for manpower forecasting. He classified these models into four distinct categories. The first which is a time-series projection model and deterministic in nature tends to extrapolate existing trends and restricts stochastic information by examining past behaviour and time relationship and then forecast future trend. Tessaring (2003) showed that this forecasting model has been adopted by different institutions like the Education and the Manpower Bureau of Hong Kong SAR, the Department for Education and Employment in the UK, and the Central Office of Education in Finland, to mention a few. According to Bartholomew et al. (1991), the model has some drawbacks even though it has shown to be fairly reliable, and relatively low-cost. Some of the drawbacks include the fact that it is only suitable for short-term forecasts because the structure of the forecast approach is limited. The model is also based on the assumption that *the future is a continuation of the past which leads to forecast errors during extrapolation once there is an element of discontinuity in the process.* The model does not also give insight into the underlying factors that could spring up changes in the occupational structure and the workforce requirement changes.

Nilakantan and Raghavendra (2005) also looked at the proportionality policy control aspects using Markov process where proportionality used here is in the sense of recruitment to promotion ratio. Chan et al. (2002) also developed a labor demand model based on the multiplier approach. According to him, depending on the type of project, projects will assume the same demand pattern given that each project commands the same level of demand on its labor requirements per unit amount of time spent. This work by Chan et al. (2002) agrees with previous work by Smith et al. (2000), CWDFC (2003), and Proverbs et al. (1995) who had in the past used the same multiplier approach in their work. A major setback is found in the immense amount of effort and the high cost of updating the database required to account for changes in labor and technology mix. Also, there is a huge limitation to the fixed coefficient model which are critical for providing accurate forecasts due to time delays as these coefficients are fixed for different categories of work as noted by Wong.

In their study to develop a stochastic manpower planning model under varying class sizes, Chattopadhyay and Gupta (2007) pointed out that the study of manpower systems is a matter of observation of the system of transition among various categories of members. Within the context and framework of the participating organization, this work aims to expound these transitions using a Markov process (probabilistic methods) as opposed to deterministic setup. Using stochastic modelling, Chattopadhyay and Khan (2004) studied a university manpower system on their respective job mobility and changes.

Vassilou (1976) showed in his work that Markov models are built for promotion and attrition while Lesson (1982) dealt with the methods of computing these state variables. Given that a Markov chain is a dynamic system with state variables, it makes it the unique model to use in determining the future step-wise transition over a period as well as the prediction of the size of the manpower per category as noted by Rao et al. (2012). To support this, Igboanugo and Edokpia (2014) did a Markovian study of manpower planning in the soft-drink industry in Nigeria. They employed the

Markov chain approach in analysing the manpower data collected over a period in which they ascertained that the method is "*effective as a decision support instrument for solving manpower problems in the industry*." The interpretation of results in his work stems from various probability theorems as given by Kemeny and Snell (1976) and from industry experience based on the probabilities of transitions. The transition probability vectors emanate from the method of maximum likelihood as given by Bartholomew et al. (1991) and Ching et al. (2013).

Now, planning involves dealing with future occurrences, and since we do not know the future, it becomes inappropriate to apply deterministic or normative models like the one exemplified above. The big question to ask though is to determine which is the most appropriate model to use depending on the situation on hand? If we consider systems of manpower in which staff movements between states are not deterministic (futuristic), that is, largely influenced by the personnel, then, one might argue that the Markov model will be most appropriate in answering the questions posed based on the preceding paragraph. On this premise, it will be plausible to discuss the probability of transitioning from one state to the other.

In countries like the Republic of South Africa where there is a shortage of skilled manpower in some quarters, it makes much sense to begin to look into manpower forecast in those areas of business needs. This will especially look into the uncertainty of attrition, knowing what could lead to it, and proffer solutions on how to curb it.

2.9 Conclusions and introduction to next chapter

The relevant literature in this chapter has been reviewed, especially in discussing the applicable theory; and also discussed the literature benchmarks. The next chapter focuses on the research design and method applied in this study.

CHAPTER THREE RESEARCH METHODOLOGY

3.1 Design

Yin (1989:29) stated that research design has to do with the logical problem as opposed to a logistical problem. Its logical nature ensures that the evidence collected enables us to answer the listed research questions. The design in this research is correlational as in case-control study or better still, trend study with the aim of making predictions based on the available data and statistical analysis. The trends can be highlighted and the statistical analysis will enable us to draw a reasonable conclusion.

The research design type is that of analysis of existing numeric data based on sufficient secondary data from the participating organization coupled with statistical modeling availability. This also points to the rationale of the study. The data required here is the existing data of the selected organization, and we will show in the subsequent sections the methods by which the data were collected and analysed.

3.2 Approach and Methodology

Hussey and Hussey (1997) defined empirical research as data based on experience or observation as a way of knowledge gain. This research is non-empirical due to the nature of the subject matter. Qualitative methods have been developed in the past to enable researchers in the social sciences domain to study associated phenomena. However, both quantitative and qualitative methods of data collection were employed in support of each other as numerical data is used. The approach adopted in this research is purely independent and objective as the researcher has no influence on the outcome.

A background literature review of general and Markov models, Markov chain theories, and manpower planning is presented. Limitations of the modelling methods are also examined. Markov chain application is introduced and based on the background material. The study is done using matrix computations to ascertain the proposition. The remainder of the research work focuses on the comparison and discussion of approaches and results. The research concludes with presentation and assessment of the results.

3.2.1 Methods

The earlier chapter presented some theories upon which the manpower planning issues of the selected organization under study is based. The method used here is in the establishment of ***states*** to be used in the study. The meaning of the ***states*** has been defined in the terminology section in chapter 1. However, in detail, it represents the position or incumbent status which an object of a Markov chain, in this case, a member of staff finds themselves during the process of transitioning towards the desired stage of retirement, with all things being equal. The Bayesian probability determination method was employed in ascertaining the probability of the step-wise movements. Therefore, in the context of the organization under study and for mathematical convenience, the seven states under study as stipulated in the *Objectives* section are:

(a) Termination (Tn)
(b) Resignation (Rn)
(c) Retirement (Rt)
(d) Staff Strength (Sst)
(e) Promotion (P)
(f) Training (T)
(g) Employment (E)

As will be presented later and stated in the objectives, we also portrayed the probabilities in alignment with the canonical form of the transition theory of the Markov chain.

It is assumed that if ***T*** is defined such that:

T = $\left[\frac{(U|O)}{(R|Q)}\right]$ Canonical form of Transition Probability Matrix (TP_rM)

Then,

$$T^n = T = \left[\frac{(U \quad | \quad 0)}{((U-Q)^{-1})R|Q^n \Rightarrow 0)}\right] \quad \ldots\ldots (12)$$

(Kemeny and Snell 1976)

Provided ***n*** is sufficiently large, where ***n*** represents the number of transitions. Proof of this theorem will be provided because the entire concept of Markov transition is built around it.

From *(1):*

F = $(U-Q)^{-1}$

And **G = FR**, where matrix variables are as defined earlier.

The Markov chain model was then used to analyse the data collected from the selected organization using non-proportional and purposive sampling methods. This is followed by requisite matrix computation of the manpower data, and the distribution of frequencies of data occurrence gives the evaluations of the transition probabilities.

3.3 Data Collection Method

The method used in collecting data was through a formalized process. We made formal requests to the appropriate channels well in advance. Several appointments were made initially with the relevant stakeholders in the participating organization. These include the data owners (HR Executive), and the HR managers.

At the first meeting, we gave a brief rundown of the research project and the topic; and also why we have decided to choose them as the selected organization. The stakeholders were advised that the data provided will be kept confidential as it involves employee details, even though names of employees were not disclosed. The consent form was then signed and is found in **Appendix A**.

In the subsequent sessions, discussions were held with the relevant Information Technology (IT) professionals assigned by the data owners to provide the relevant data as requested.

3.4 Validity of Data

The data used in this research were from very reliable sources. The sources of data are also verifiable. The participant organizations have reliable databases, hosted by Oracle and manned by competent IT professionals.

3.5 Originality and Limitations

I, at this moment, declare that this minor dissertation is my unaided work. It is being submitted for the degree of Master of Philosophy in Engineering Management to the University of Johannesburg. It has not been presented or submitted previously for any degree or examination in any other University.

3.6 Ethical Consideration of the Research

Ethics clearance was required by the participant organization from which the data was gathered. The consent letter sent stipulates that participation in the study is intentional and voluntary; that the data will be treated in a private manner; that the participants will remain anonymous (where applicable), and that their privacy will be respected. We also declare that the standard practice of ethical professionalism will be upheld in the proposed research task. I undertake to bring to the attention of the Faculty Ethics Committee any deviations to this work which may affect ethical matters about this project.

3.7 Summary and Introduction to next chapter

Chapter 3 discussed the methods used in this research project. Secondary data was also acquired through a qualitative method of structured meetings with relevant stakeholders.

The next chapter focuses on the results and analysis of these results, and will obviously point to the successful method used.

CHAPTER FOUR RESEARCH FINDINGS

4.1 Presentation of Data

In the previous chapter, we discussed the method by which we hope to actualize our objectives and aims. The areas of manpower which form the data set are:

(a) Termination (Tn);
(b) Resignation (Rn);
(c) Retirement (Rt);
(d) Staff Strength (Sst);
(e) Promotion (P);
(f) Training (T); and
(g) Employment (E).

Vajda (1978), Bartholomew (1976), Raghavendra (1991), and most recently, Igboanugo and Onifade (2011) adopted a mathematical and statistical approach to dealing with the Markov model for manpower planning using the following notation:

- ***t*** = (1, 2, 3….T) where T = 9 in this case and the timeframe is measured in years;
- ***i, j*** = 1, 2, …, K: representing the manpower states under study and the Markov object transitions from state ***i*** to state ***j***;
- $N_j(t)$ = number of Markov objects at state *j* at the start of period ***t***; and
- $Pr_{ij}(t)$ = probability that a Markov object (staff) in state ***i*** at the beginning of period *t* is in state *j* at the start of the next period.

We tend to follow the same sequence and consequently, it follows that:

$N_1(t)$ = number of Termination;
$N_2(t)$ = number of Resignation;
$N_3(t)$ = number of Retirement;
$N_4(t)$ = number of Staff Strength;
$N_5(t)$ = number of Promotion;
$N_6(t)$ = number of Training; and
$N_7(t)$ = number of Employment.

4.2 Case study example: The Manpower Data Set

The data set of states under consideration is shown in the table below. This is a 9 – year manpower data obtained from the organization under study. It is also imperative to note that the Markov object (members of staff) under consideration is of permanent status. No contract or temporary staff is considered. The annual breakdown of figures for these states is presented in the appendix section.

Table 1: Manpower data with state space

No	**Manpower states**	**Manpower numbers**
1	Termination	1549
2	Resignation	1309
3	Retirement	467
4	Staff Strength	18089
5	Promotion	1642
6	Training and development	9910
7	Employment	1164
Total		**34130**

4.3 Assumptions applied in the computation

Some basic assumptions were upheld given the dataset as adopted by Igboanugo and Onifade (2011). It makes sense also that:

- No retired staff can go for training, or be promoted, or be terminated, or resign, or be employed, or still be part of the staff strength;
- No terminated member of staff can be retired, or go for training, or be promoted, or resign, or be employed, or still be part of the staff strength;
- No resigned member of staff can be retired, or go for training, or be promoted, or be terminated, or be employed, or still be part of the staff strength;
- A retired, or resigned, or terminated personnel remains at that state;
- A newly employed staff cannot go into retirement;
- A recently promoted staff cannot be terminated or retired; and
- A member of staff undergoing training and development cannot be terminated or retired.

These are attrition states. It means that the probability of moving from these states to another is not possible, and hence, said to be zero. Consequently, it follows mathematically, Igboanugo and Onifade (2011), that:

$P_{r11} = P_{r22} = P_{r33} = 1;$

$P_{r12} = P_{r13} = P_{r14} = P_{r15} = P_{r16} = P_{r17} = 0;$

$P_{r21} = P_{r23} = P_{r24} = P_{r25} = P_{r26} = P_{r27} = 0;$

$P_{r31} = P_{r32} = P_{r34} = P_{r35} = P_{r36} = P_{r37} = 0;$

$P_{r51} = P_{r53} = 0;$

$P_{r61} = P_{r63} = 0;$ *and*

$P_{r73} = 0;$

4.4 The Analytical Framework: Transition Probabilities

We present in this section, the theory on which the manpower transition by way of the defined states is based. We fit the Markov chain model into the record of manpower transition over a 9-year period obtained from one of the prestigious South African Universities under consideration.

Given that a newly employed personnel commences movement from initial distribution d_i, and traverse among the seven states, the stabilized transition distribution $\boldsymbol{T_s}$, of this employee at the 9th year for $\boldsymbol{n}$ = 9 years as in equation (13) below according to Igboanugo and Onifade (2011) is given by

$$\boldsymbol{T_s = d_i T^n} \quad \text{.......} (13)$$

Where $\boldsymbol{d_i = R}$, an arbitrary matrix concerned with the transition from non-absorbing to absorbing states and the output of the transition matrix $\boldsymbol{T}$ tends to be regular or rather stationary after $\boldsymbol{n}$ transitions.

4.4.1 Proof

We show that, given the canonical form of the transition probability matrix,

$$\text{T} = \left[\frac{(U|O)}{(R|Q)}\right] \quad \text{.......} (14)$$

According to Kemeny and Snell (1976) then,

$$T^n = \text{T} = \left[\frac{(U \quad | \quad O)}{((U-Q)^{-1})R|Q^n \Rightarrow 0)}\right] \quad \text{.......} (15)$$

The component, $(U-Q)^{-1})R$, is a very important function as it gives the solution matrix. As stated earlier, the fundamental matrix points to the probable or apparent number of visits to respective states before attrition occurs. At initial state $\boldsymbol{i}$, the chain visits state $\boldsymbol{j}$ in one step. The next step, given by the elements of the $\boldsymbol{i^{th}}$ row and $\boldsymbol{j^{th}}$ column of matrix Q, represents the apparent number of visits made by the chain. Following this trend, therefore, the corresponding entry in the matrix Q^2 gives the anticipated number of times the chain visits state ***j*** in two steps. Thus, from equation (14) Kemeny and Snell (1976),

$$T^2 = \left[\frac{(U|O)}{(R|Q)}\right]$$

$$= \left[\frac{(\quad U \quad | O)}{((U+Q))R| \ Q^2)}\right]$$

$$T^3 = \left[\frac{(U|O)}{(R|Q)}\right]^3$$

$$= \left[\frac{(\quad I \quad | \quad O)}{((U+Q+Q^2))R| \ Q^3)}\right]$$

$$T^4 = \left[\frac{(U|O)}{(R|Q)}\right]^4$$

$$= \left[\frac{(\quad U \quad | \ O)}{((U+Q+Q^2+Q^3))R| \ Q^4)}\right]$$

.

.

.

$$T^n = \left[\frac{(U|O)}{(R|Q)}\right]^n$$

$$= \left[\frac{(\qquad U \qquad | \quad O)}{(U+Q+Q^2+Q^3+Q^4+Q^{n-1}))R \mid Q^n)}\right] \quad \text{... (16)}$$

According to Kemeny and Snell (1976), in equation (16), the infinite sum is of the form, $U + Q + Q^2 + Q^3 + Q^4 + \ldots$ which when multiplied by (U - Q) is the same as $(U - Q)^{-1}$, the fundamental matrix, therefore,

$(U + Q + Q^2 + Q^3 + Q^4 + \ldots)(U - Q) = U + Q + Q^2 + Q^3 + Q^4 + \ldots - Q - Q^2 - Q^3 - Q^4 + \ldots = U$

and this verifies the proof.

P_{rij} is greater than zero, but less than 1 and Q^{n+1}/Q^n is also less than zero, therefore, the series in equation (12) in $U + Q + Q^2 + Q^3 + Q^4 + \ldots + Q^{n+1}$ is geometrically progressive and decreasing monotonically with absolute convergence; hence, as $n \rightarrow \infty$, $Q^n \rightarrow O_k$ which is a ***k x k*** zero matrix, and therefore, equation (17) Kemeny and Snell (1976) gives

$$T^n = \left[\frac{(U\,|O)}{(FR|O_k)}\right] \ldots\ldots (17)$$

The ***FR*** section of equation is the solution matrix which gives the probability that the system is absorbed in one of the absorbing states given that it was initially in a non-absorbing state

4.5 Computation of Matrix of Transitional Probabilities: The Bayesian Method

The calculations for the state probabilities for the absorbing and non-absorbing states are shown below using the Bayesian method of probabilities which is simply a ratio obtained from the *manipulation of conditional probabilities*. We adopt the same approach by Igboanugo and Onifade (2011) in the computation of state probabilities.

The general form of the Bayesian probability (Olshausen 2004) is given by

$$Pr\,(H|D) = \frac{Pr\,(D|H)Pr\,(H)}{Pr\,(D)} \ldots\ldots (18)$$

which can also be displayed as a summation as shown below. The mathematical detail is not included in this work.

In general, the summation in terms of ***Pr_{ij},*** equation (19) according to Bartholomew et al. (1991), is given by

$$Pr_{ij} = \frac{N_i(t)}{\sum_j^k N_j(t)} \ldots\ldots (19)$$

4.5.1 Transition from state 4 to other states - (Movement of the entire workforce)

Following the assumptions in section 4.3, our first transition starts from state 4 to other states simply put as Pr_{4j}. Given that all $Pr_{ij} > 0$, as this applies to the staff strength, then, this points to *ergodicity* of the manpower transition.

Therefore,

$$\sum_j^k N_j(t) = 1549 + 1309 + 467 + 18089 + 1642 + 9910 + 1164 = 34130$$

Hence,

$Pr_{41} = \frac{1549}{34130} = 0.0454$ rounded off to 4 decimal places and following the same method, we obtained in similar way the probability of movements starting from state 4 to other states and consequently,

$Pr_{42} = 0.0384$

$Pr_{43} = 0.0137$

$Pr_{44} = 0.5300$

$Pr_{45} = 0.0481$

$Pr_{46} = 0.2904$

$Pr_{47} = 0.0341$

4.5.2 Transition from state 5 to other states - (Movement of newly promoted staff)

A promoted staff can find oneself in any of those states in *n* different steps during transition except for states 1 and 3 respectively following the assumptions made in section 4.3. The denominator of the equation changes as we expunge values of $N_j(t)$ which meets a particular condition.

Since no new promoted staff can go into retirement or be terminated, it follows that $N_j(t) = N_1(t) = 0$ and $N_j(t) = N_3(t) = 0$ which means that the values of 1549 and 467 are expunged respectively for the determination of the denominator.

Therefore,

$$\sum_j^k N_j(t) = 0 + 1309 + 0 + 18089 + 1642 + 9910 + 1164 = 32114$$

Hence,

$Pr_{51} = \frac{0}{32114} = 0.000$ rounded off to 4 decimal places and following the same method, we obtained in similar way the probability of movements starting from state 4 to other states and consequently,

$Pr_{52} = 0.0408$

$Pr_{53} = 0.0000$

$Pr_{54} = 0.5633$

$Pr_{55} = 0.0511$

$Pr_{56} = 0.3086$

$Pr_{57} = 0.0362$

4.5.3 Transition from state 6 to other states - (Movement of staff in training and development)

Manpower training and development is a serious matter in the organization under study; hence, the assumption that no member of the manpower system under training and development are retired or terminated. This also implies that a member of staff can transit from one state to another in ***n*** different steps during transition except for states 1 and 3 respectively following the assumptions made in section 4.3. Therefore, we expunge their respective values of $N_j(t)$ to determine the denominator of the equation.

It also follows that since no new members of staff who are in training and development can go into retirement or be terminated, it follows that $N_j(t) = N_1(t) =$

0 and $N_j(t) = N_3(t) = 0$ which means that the values of 1549 and 467 are expunged respectively for the determination of the denominator of the equation.

Therefore,

$$\sum_j^k N_j(t) = 0 + 1309 + 0 + 18089 + 1642 + 9910 + 1164 = 32114$$

Hence,

$Pr_{61} = \frac{0}{32114} = 0.000$ rounded off to 4 decimal places and following the same method, we obtained in similar way the probability of movements starting from state 4 to other states and consequently,

$Pr_{62} = 0.0408$

$Pr_{63} = 0.0000$

$Pr_{64} = 0.5633$

$Pr_{65} = 0.0511$

$Pr_{66} = 0.3086$

$Pr_{67} = 0.0362$

4.5.4 Transition from state 7 to other states - (Movement of newly employed staff)

Generally, much costs are incurred during the hiring or recruitment process in most organizations, like the case study organization, hence, the assumption that no newly employed member of staff can go into retirement; however, a recently employed member of the manpower system can transit to other states in ***n*** different steps. Therefore, given the assumptions made in section 4.3 on newly employed members of staff, we expunge the value of $N_j(t)$ for retirement to determine the denominator of the equation.

Hence, it follows that $N_j(t) = N_3(t) = 0$ which means that the value of 467 is expunged respectively for the determination of the denominator of the equation.

Therefore,

$\sum_j^k N_j(t) = 1549 + 1309 + 0 + 18089 + 1642 + 9910 + 1164 = 33663$

Hence,

$Pr_{71} = \frac{1549}{33663} = 0.0460$ rounded off to 4 decimal places and following the same method, we obtained in similar way the probability of movements starting from state 4 to other states and consequently,

- $Pr_{72} = 0.0389$;
- $Pr_{73} = 0.0000$;
- $Pr_{74} = 0.5374$;
- $Pr_{75} = 0.0488$;
- $Pr_{76} = 0.2944$; and
- $Pr_{77} = 0.0346$.

4.6 The Transition Probability Matrix Space (TPrMS)

Table 2 gives the transition probability matrix based on the computations done in the previous section; where the vertical components represent the "now", and the horizontal components represent the "future".

Table 2: Transition probability matrix

State Nomenclature		*1*	*2*	*3*	*4*	*5*	*6*	*7*
		Termination	*Resignation*	*Retirement*	*Staff Strength*	*Promotion*	*Training*	*Employment*
1	*Termination*	1	0	0	0	0	0	0
2	*Resignation*	0	1	0	0	0	0	0
3	*Retirement*	0	0	1	0	0	0	0
4	*Staff Strength*	0.0454	0.0384	0.0137	0.5300	0.0481	0.2904	0.0341
5	*Promotion*	0.0000	0.0408	0.0000	0.5633	0.0511	0.3086	0.0362
6	*Training*	0.0000	0.0408	0.0000	0.5633	0.0511	0.3086	0.0362
7	*Employment*	0.0460	0.0389	0.0000	0.5374	0.0488	0.2944	0.0346

Table 3:Absorbing states and fundamental Matrix

U			***O***			
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0454	0.0384	0.0137	0.5300	0.0481	0.2904	0.0341
0.0000	0.0408	0.0000	0.5633	0.0511	0.3086	0.0362
0.0000	0.0408	0.0000	0.5633	0.0511	0.3086	0.0362
0.0460	0.0389	0.0000	0.5374	0.0488	0.2944	0.0346
R			***Q***			

4.7 Results

4.7.1 The Habituation Diagram

The diagram below in Fig 3 shows the transition of staff members from state to state given the state spaces S = {S_1, S_2, S_3, S_4, S_5, S_6, S_7} equivalent to S = {S_a, S_b, S_c, S_d, S_e, S_f, S_g} as will be used later. The states of entrapment or absorption are clearly seen in states 1, 2, 3 as indicated in the form of a closed loop. As indicated by a "no-arrow-going-out" from these states but only going in.

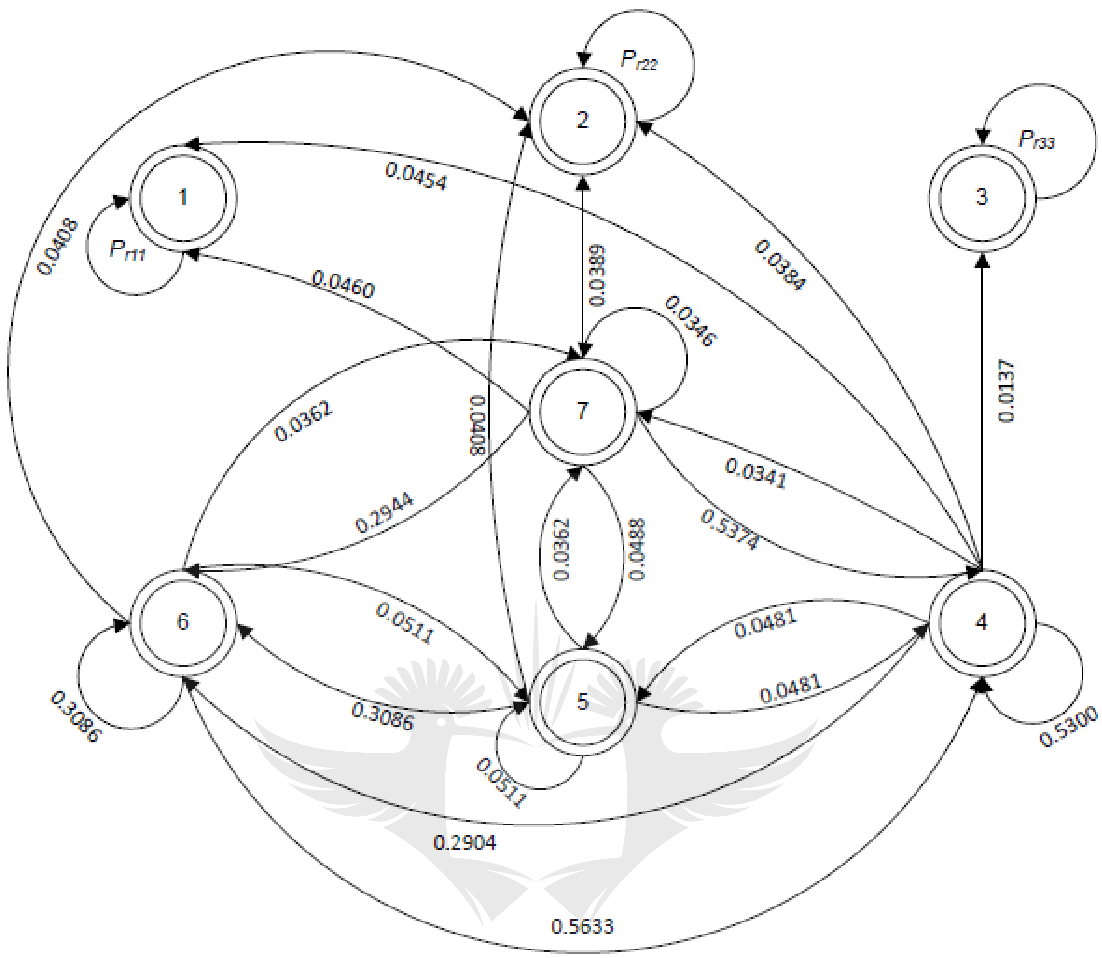

Figure 3: The Habituation Diagram

This means that it is a state in which once entered, it is impossible to come out, and this clearly defines a Markov chain process and possesses Markov chain properties. Other states 4, 5, 6, 7 simply indicate the step-wise movement of staff members across these states before absorption in the absorbing states.

The various computations in the following section below focuses on the most significant results that centres on the mean and variance of transition levels among transient states. This is also founded based on the canonical form of the long-run TP_rM in the matrix form shown in section 4.4.1 as:

Table 4: Absorbing states and fundamental Matrix

$$\left[\begin{array}{c|c} U & O \\ \hline (U - Q)^{-1} R & Q^{n} \end{array}\right] = \left[\begin{array}{ccc|cccc} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \hline 0.3840 & 0.5072 & 0.1096 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.3598 & 0.5390 & 0.1019 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.3598 & 0.5390 & 0.1019 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.3893 & 0.5143 & 0.0973 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{array}\right]$$

as $Q^n \to 0$.

4.8 Quantities of the Absorbing Chain Matrix and their respective computations

4.8.1 Average Visitation (AV)

This is given by the fundamental matrix (F) which specifies the mean number of periods that staff in their initial ***i***th non-absorbing state habituate to ***j***th non-absorbing states before absorption. Also, the summation of the entries in the ith row of the fundamental matrix gives the mean habituation among states by members of staff in the ***i***th non-absorbing state before absorption. Hence, we compute with MATLAB, the fundamental matrix. The average visitation, (AV) is given by $F = (U - Q)^{-1}$.

4.8.1.1 Computation of the Fundamental Matrix

Details of the computation of the fundamental matrix are shown in Appendix B. And from earlier definition of Q, a random matrix depicting the transient to transient states movements,

$F = (U - Q)^{-1}$

Using MATLAB computation, we deduce the inverse of the matrix (U - Q) which is also shown in Appendix B.

We shall unwrap the various meanings embedded in the different matrices in the discussion section of this work.

4.8.2 The Resolution Matrix of the Absorbing Markov Chain

The resolution matrix ***G = FR***, is an essential quantity of the absorbing chain. Not only does it point to the long run distribution of the workforce, but it also specifies the probability estimation of staff habituation to any absorbing state given that they emanated from any of the non-absorbing states.

4.8.2.1 Computation of the Resolution Matrix G

Specifying parameters, the initial distribution d_i = **R**, an arbitrary matrix which gives the movements of staff from non-absorbing states to states of absorption.

In computing the resolution matrix G, there is a need to introduce the fundamental matrix F, as it gives the average steps staff take before transiting into absorption. Therefore, ***G = FR.***

Computations with MATLAB, again, give the solution matrix G. All computations for the resolution matrix is shown in Appendix C.

4.8.3 Variances in the Absorbing Chain

Here, different quantities are expressed in terms of the fundamental matrix. We consider two different variances in this absorbing Markov Chain. First, we look at the variance of the mean of transition (F_v) where (F_v) is an ***NxN*** matrix or simply put, the variance of the fundamental matrix; and the variance μ_v, which gives the variance of transition among non-absorbing states.

So, the variance tends to explain how reliable or unreliable the *means* are for the Markov chain. For example, if the variance of the fundamental matrix (F_v) is very large compared to its square, we can conclude that the means are not fairly reliable as concluded by Kemeny and Snell (1976:50).

The mathematical methods which are not shown, and the various equations (20), (21), (22), and parameters used in computing these variances are found in Kemeny and Snell (1976: pp 49 - 52).

4.8.3.1 Computation of Variances

The variance of the fundamental matrix is given by

$F_v = F\,(2F_{dg} - U) - F_{sq}$ .... (20)

and the variance of transition

$\mu_v = (2F - U)\,\mu - \mu^2$.. (21)

where these parameters have been defined in section 1.8.2.

First, we find the diagonal matrix of the fundamental matrix F_{dg} which is simply gotten by keeping the diagonal values of the fundamental matrix and assigning other elements of the matrix a zero value. Then, we find the square of the fundamental matrix F_{sq} and the variance of the fundamental matrix given by $F_v = F\,(2F_{dg} - U) - F_{sq}$

In the same vein, we compute the variance of the transition among non-absorbing states. This component column vector μ_v, is given by $\mu_v = (2F - U)\,\mu - \mu^2$. The other component column vector μ**,** specifies the directional movements and is also given by

$\mu = F\,v$ .... (22)

Where v is an eigenvector given by:

$$v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$ and we already know F, the fundamental matrix.

The other component column vector μ, which specifies the directional movements above resonates the popular eigenvalue problem scenario of the type $Bz = \lambda z$ where B = Fundamental matrix F, and z represents the transpose of an eigenvector and λ denotes the eigenvalue. Mathematically,

$B = F$;

$z = [1,1,1,1]^T$

λ = eigenvalue.

This implies that:

$Bz - \lambda I z = 0$

$(B - \lambda I)z = 0$ which implies that $F v = \lambda v$

All computations are given in Appendix D.

4.9 Analysis of Results

Following the discussions and calculations that ensued in the previous sections, we embark on interpreting these results with the sole aim of achieving the set objectives of this project. The analysis focuses specifically on three key areas which are the:

- fundamental matrix F;
- its offshoot and the solution matrix G; and
- and μ, a column vector, that specifies the expected number of transitions among non-absorbing states.

4.9.1 Interpretation of the Fundamental Matrix F

From section 4.1, we designated the different states and specified the last 4 states designated d, e, f, g as the non-absorbing states given that state space S = $\{S_a, S_b, S_c, S_d, S_e, S_f, S_g\}$ is equivalent to state spaces S = $\{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$ as mentioned in section 4.7. These states will be used to interpret the fundamental matrix. All records in the fundamental matrix are positive and tend to shed light on the average visitation or transition among staff members in the transient states.

We now commence the interpretations of the movements of staff as follows:

4.9.1.1 Transition of members within the states of no absorption

(i) d to d with an input of 8.0018 (determination): This range pertains to members of staff who stays put in the system and migrate to different job positions. Such members, on the average, change their positions eight times subject to variance. This estimate varies for 1 to 15 given a deviation of approximately 7, that is, 8±7.

(ii) d to e with an input of 0.6354: On the average, 6 in 10 permanent staff members get promoted within the period under consideration.

(iii) d to f with an input of 3.8362: This represents the number of times a permanent or full-time member of staff goes to some form of training within the 9-year period. This is subject to a deviation of 4 which implies that it could happen for up to 8 times within that time limit.

(iv) d to g with an input of 0.4503: means that 4 in every 10 permanent staff is subject to a review within this period. This shows that this is a very high demanding environment where high performance is required.

(vi) e to e with an input of 1.6752: implies that a promoted staff member remains in the promoted state

(vii) e to f with an input of 4.0768: tells us that a promoted staff member goes to at least 4 different training before the next promotion. This can be up to 8 times based on the standard deviation of 4.

(viii) e to g with an input of 0.4785: implies that 4 in every 10 promoted staff member undergo a review for non-performance.

(ix) f to d with an input of 7.4409: suggests that staff members who are on training, even a training leave, on the average, return 7 times to carry out some assignments. However, this is subject to a standard deviation of 7 which means that staff member may have from zero (non-appearance) to 14 of such transactions while on training depending on the category of staff.

(x) f to e with an input of 0.6752: implies that more than 6 in 10 members of staff who undergo training get promotion sooner rather than later.

(xi) f to f with an input of 5.0768: suggests that members of staff who are in training are scheduled to undertake between 1 to 9 different training based on a standard deviation of approximately 4.

(xii) f to g with an input of 0.4785: implies that 4 in every 10 staff members in training still undergo some form of review.

(xiii) g to d with an input of 7.0995: suggests that a newly employed staff member does an average of nil to 14 visits to different job positions before being confirmed with a standard deviation of 7. This actually depends on the peculiarity of the position.

(xiv) g to e with an input of 0.6443: implies that 6 in every 10 staff members get promoted soon after being hired depending on the nature of the position.

(xv) g to f with an input of 3.8897: indicates that a newly employed staff could be scheduled for about 4 different kinds of training. However, with a standard deviation of 4, it shows that this could happen from nil to 8 different kinds of training depending on the position under consideration.

(xvi) g to g with an input of 1.4566: suggests that a newly employed staff makes on the average one visit to another position/department while remaining or maintaining his primary position.

4.9.2 Interpreting the Solution Matrix G

This gives us the probability that a staff member will go into one of the absorbing states having started from one of the non-absorbing states.

(i) d to a with an input of 0.3840: implies that there is a 38% probability that a permanent staff's contract could be terminated.

(ii) d to b with an entry of 0.5072: suggests that more than half of the members of staff is difficult to be retained. This could be as a result of people seeking greener pastures.

(iii) d to c with an input of 0.1096: suggests that only about 10% of staff go into retirement within a 9-year period.

(iv) e to a with an input of 0.3598: implies that more than 64% of promoted staff members retain their job and do not get terminated.

(v) e to b with an input of 0.5390: suggests that more than half of the promoted staff members tend to resign.

(vi) e to c with an input of 0.1019: points to the fact there is a 10% chance that a promoted staff would retire.

(vii) f to a with an entry of 0.3598: suggests that more than 35% of staff members trained tend to get terminated. (An area of concern basically for all states going into termination)

(viii) f to b with an input of 0.5390: refers to the point that approximately 53% of trained staff members or staff members in training go on to resignation.

(ix) f to c with an input of 0.1019: that there is a 10% chance of a staff member in training retiring.

(x) g to a with an entry of 0.3893: suggests that there is a 38% chance that a newly employed staff could be terminated. This could also include staff on probation and/or due to performance related issues.

(xi) g to b with an entry of 0.5143: implies that about 51% of recruited staff tend to resign after been hired.

(xii) g to c with an input of 0.0973: tells us that there is more than 90% chance that a newly employed staff member will not retire.

4.9.3 Cumulative transition of staff among non-absorbing states, μ

The column vector, μ, has the following matrix as obtained in equation 22:

$$\mu = \begin{bmatrix} 13.0000 \\ 13.0000 \\ 13.0000 \\ 13.0000 \end{bmatrix}$$

This implies that regardless of the i^{th} state (start state), all staff members tend to wander across the non-absorbing states up to 13 times prior to absorption. This is subject to a standard deviation of 12.68 given a variance of 161 which results to a range of nil to 26 times.

4.10 Discussion of Results

Having analysed the results which focus on the promises of the approach as made earlier, we discuss and summarize our findings below with the sole aim of achieving the purpose of the study and to answer our impending research questions effectively.

Flowing from the analysis, we see that for several motives stretching from the institution's talent retention policy, economic migration, wastage, career change; approximately 37% of the staff members in states d, e, f, g are terminated. On the other hand, approximately 52% of staff members who are in any of the non-absorbing states tend to resign from the University system. This is a cause for concern as it shows some level of consistency for these two categories (termination and resignation). The high percentage value seen under "resignation" basically points to a job change. Conclusions that can be drawn include but not limited to better pay, career actualization, and new opportunities.

Markedly, the wastage rate via retirement is low by all means sitting at approximately 10% low. This supports the fact that not many permanent staff members stay through to retirement.

As compared to the work by Igboanugo and Onifade (2011) in which he proclaimed that the policy of the University system that he considered was liberal, such is not the case in this study. We can see, evidently that the system is ***rigid and stern*** in its broadest sense based on the consistency of results obtained. The discussions of the research questions support the claim.

4.10.1 Discussion of Research Question 1

Research question one pertains to how much manpower flow to termination, resignation, and retirement (wastage) respectively as absorbing states. Wastage is a form of attrition that could result from death, retirement, resignation, termination, and the likes.

According to Ogbogbo et al. (2013), the human resource is an unpredictable entity and forms the most crucial part of an organization's entire resources for achieving its set objectives. Mythili et al. (2013) stated that most wastages stem from unhappy employees especially after appraisals which could be as a result of unfavourable organizational policies and restructuring, higher responsibilities with low pay, lack of growth opportunities, empty promises, to mention a few. As a consequence of the exodus, the business tends to suffer breakdown whenever it occurs at a continuous rate. Hence, to retain top performers at any organization, the onus lies with the organization to develop policies that are geared towards staff retention and at all times provide an environment that convinces the employees to remain in the system for an extended period of service. On the other hand, organizations could shed manpower when business is not booming. However, they need to sustain its skilled and qualified personnel to keep the business running as stated by Mohan and Ramanarayanan (2013).

The Markov model used in this work, and specifically from the column vector, μ, the cumulative transition of staff among non-absorbing states prior to absorption, sheds light on one of the key characteristics of the manpower policy for the organization under study. This is that regardless of the start point, each and every member of staff tend to transit across the non-absorbing states before going into complete absorption from zero to thirteen positions. Therefore, it suffices to say that employed staff members are used judiciously and probably stay for an extended period before hitting the exit door.

Flowing from the analysis done earlier, approximately 37% of manpower flow to termination, 52% of manpower flow to resignation, and 11% of manpower flow to retirement. This could be as a result of one of the reasons outlined in the preceding paragraphs.

4.10.2 Discussion of Research Question 2

This research question relates to how often do members of staff get trained and promoted. Skulj (2008) stated that, during employment, some skills are needed to carry out assigned duties and responsibilities and are usually acquired through some form of special training as a shortfall in the skill set of the employee could result in inefficiency.

Promotion is a system of elevation from one position to a higher one regarding job title. It could also be in the form of an increase in perks and total emoluments. This is achieved based on recognition of hard work, evidenced by required work outputs. It could also come as a result of gap-filling of positions made available through loss of staff member, and the position needs to be filled immediately. Promotion can also be handed down as a result of the length of service in the organization, subject to availability of positions. It could also be acquired through an imminent need to fill a position to which there is a shortage of skillset at the level.

Regardless of the method of acquisition, Skulj (2008) stated that promotion is key to achieving the desired structure in any organization that wants to maintain stability.

Following the analysis of the computational results in section 4.9.1.1, sub-section (ii) and (iii) which shows the average visitation for these states, one can see that the University policy on training and promotion is ***generous***; given that 6 in every ten staff members get promoted and that permanent staff members go to some form of training up to 8 times depending on their job specifications. This gives another major characteristic of the manpower policy of the organization stemming from the Markov analysis.

4.10.3 Discussion of Research Question 3

Research Question 3 focuses on the relationship between lack of training and promotion, and wastage. Basically, it begs the question: Does the lack of training and promotion lead to any form of attrition?

Considering the discussion in the preceding research questions, it would suffice to say that a lack of promotion and training will lead to some form of wastage. However, based on the answers furnished for research question 2, we can assert that there is no lack of training and promotion in the system. The system is quite liberal in these regards as discussed above. The mind-boggling issue, which is the reverse case and will need further review, is that there is also a high rate of attrition flowing from these two areas which are areas of great concern. This implies that there are employees who leave regardless of getting promoted and trained. This might point to other restructuring policies and overall organizational attitude towards staff members.

4.11 Conclusion of chapter

The chapter commenced with the presentation of data and followed by computation of various quantities needed to do the required analysis. From the analysis done, we can make some assertions and answer the research questions asked. The next chapter is concerned with drawing reasonable conclusions based on the information set.

CHAPTER FIVE CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This research work assessed the stock and flow progression. That is, the probability of staff transition from one position to another, staff retention level, the presence of structures critical to staff's job satisfaction; and the effect of organizational policy towards overall staff development using the popular and effective Markov chain model in the data analysis. The fundamental need for modelling manpower, irrespective of the technique adopted or used variables as Walker (1971) stated is to increase the grasp of the subject of manpower stocks and flows by manpower managers and stakeholders towards manpower planning improvement.

Models, in general, should give insight to information that could not have been considered in the planning of manpower such as job transition patterns, individual aspirations, and general behaviours of the workforce. This information should be considered in the development of models for predicting manpower behaviour in an organization as in Walker (1971). The Markov model exhibited a significant result that is consistent with its theory, having being marked as the better model for such predictions. Through its interpretation, one gains insight into the hidden details embedded in large manpower data which ultimately guides manpower managers and other stakeholders in making informed decisions.

In their article titled, a manpower model for small groups, Davies and Howard (1981) stated that the prediction and the control of transitions or stationarity of the workforce in an organization depends on one's ability to have a full grasp of the nature of reasons for the transitions and stationarity. Having reviewed all other models which are very beneficial in order aspects of manpower resources, the Markov chain approach is more suited to the prediction of future behaviour of the workforce given that the transition of manpower within and without the system is difficult to control.

The Markov chain model, using the matrix transformations and its interpretations is simple to comprehend, and gives a realistic view of where the organization stands in terms of its workforce movements and the reason behind it. This also makes it appear to meet several requirements for a robust system for the prediction of manpower behaviour.

Given the analysis and discussions done in the previous chapter, we conclude that there is a significant and considerable manpower loss in the system through the two of the three wastage channels. Most importantly, the Markov chain model used did shed a lot of light towards unearthing the hidden trends in the manpower planning data and thereby proffering a suitable solution towards aligning manpower policies to corporate goals. The transition probability matrix which was employed exhibited Markov properties of stochastic regularity. The most important quantity which is the "Solution Matrix, G = FR," presented us with a trustworthy solution for manpower predictions coherent with the Markov chain theory as the base model.

It is important to note that models used for forecast cannot be validated until the arrival of the forecasted future. The study validates the effectiveness of Markov chain model by its ability to expose hidden details in manpower data, thereby showcasing the existing manpower policies which can be used to predict the future behaviour through the results and its interpretation.

As discussed in previous chapters, many manpower planning models exist. However, the Markov chain model appears to be more intuitive. Kim and Smith (1989) had identified Markov model as the most potent tool for the analysis of complex stochastic systems and this flexibility of the Markov chain model has been widely adopted. This tool especially its ability to incorporate manpower data in forecasting future behaviour through the analysis of current transition probabilities.

Compared with System Dynamics (SD)model, which is one of the computer simulation techniques, the SD model shows causes of undesired behaviour and revelation of feedback loop structures (DoD 2005). However, an advantage of the Markov analysis is that it allows for quick impact assessment of various manpower policies, in other words, it can point to policy changes regarding training, promotion and ultimately, staff retention. It is also very intuitive especially in relation to establishing the probability of step-wise movement from one position to another. Markov models are also easily integrated with other models like the programming models for optimization. Furthermore, potential limitations of the SD model (qualitative and quantitative) stems from the lack of formal procedures in the conceptualization diagramming of the real situations comprising stock and flow variables. The lack of formal procedures in conceptualization is a big concern for new entrants as noted by (DoD 2005). Notwithstanding the effect of the lack of formal procedures in conceptualization, modelling of soft variables like suitability of environment, employee morale and motivation, and job satisfaction casts doubt in the modelling result because they are hard to measure.

The most important question according to (Price et. al 1980) to ask is which model is most suited to a particular situation. The approach used in this study which speaks to the proportion of individuals moving from one state to the other ties to the observation of (Price et. al 1980) in which he believes that given that staff transitions are probabilistic and cannot be controlled, the Markov approach is best suited for such systems. It is the most straight- forward, easy to use and provides a simple framework for modelling staff transition based on empirical observation that flows are proportional to stocks according to Bartholomew et al. (1991). This makes it the most appropriate tool in forecasting future manpower behaviour for the organization.

The results obtained through the Markov model validates that improved effectiveness in manpower management can be gained through a thorough understanding of staff transition from one position to another, the reason behind the movements, the factors that tend to provide job satisfaction, and impact of a liberal policy while other factors remain constant. Through the results, using this model and tools which it offers has not only helped us to see the current trend, but also to predict the future behavioural pattern of manpower policy in the institution under study. It is also evident that the use of the model will certainly give manpower managers a better understanding when making policies and decisions towards staff retention geared towards the achievement of organizational goals.

5.2 Recommendations for Future Work

During the analysis phase, and based on the information that emanated, the researcher is faced with a couple of questions which need further research and investigations. These questions relate to the level of attrition or wastage experienced through termination and retirement and include:

- Why do we have the high degree of wastage through termination and retirement? What could be the cause? Is it job dissatisfaction? Is it greener pastures like we stated above?
- Why do we have so many imbalances regarding staff members trained and promoted leaving the system?

This can be achieved through more qualitative research methods like using questionnaires, gathering data from exit interviews conducted and the likes. By answering some of these questions, the output of the research should be fed back into the system to make more informed decisions that will promote staff retention having invested in them through training and promotion.

As stated earlier, this study was limited by the overall sample size, confining us to nine years of manpower data due to the availability of verifiable data. Future studies with the Markov model would profit from a bigger data sample population, including a wider range of states. This would allow for improved and better accuracy in forecasting the future manpower behaviour towards creating a better manpower policy.

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APPENDIX A CONSENT FORM

The consent form is hereby attached. It contains the letter from the organization under study giving their permission for the release of the manpower data.

OFFICE of the Registrar

Private Bag 3, Wits, 2050, South Africa Tel: +27 11 717 1201/2 Fax +27 11 717 1217 E-mail: registrar@wits.ac.za

E-mail nita.lawton-misra@wits.ac.za
Fax 086 553 3695
Tel +27 (0)11 717-1204

12 June 2014

TO WHOM IT MAY CONCERN

"Manpower data pertaining to Retirement, Wastage, Interdiction/Suspension, Training, Staff stock, Recruitment, Severance"

It is hereby confirmed that disclosure of the above information for research purposes has been approved. Please be advised that it is your right to withdraw from participating in the process if you find the contents intrusive, too time-consuming, or inappropriate. The necessary ethical clearance has been obtained.

Should the University's internal mailing system be the mechanism whereby this questionnaire has been distributed, this notice serves as proof that permission to use it has been granted.

Students conducting surveys must seek permission in advance from Heads of Schools or individual academics concerned should surveys be conducted during teaching time.

Ms Carol Crosley
University Registrar

APPENDIX B Computation of the Fundamental Matrix

The ***MxM*** identity matrix (U) is given by

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.5300 & 0.0481 & 0.2904 & 0.0341 \\ 0.5633 & 0.0511 & 0.3086 & 0.0362 \\ 0.5633 & 0.0511 & 0.3086 & 0.0362 \\ 0.5374 & 0.0488 & 0.2944 & 0.0346 \end{bmatrix}$$

$$(U - Q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5300 & 0.0481 & 0.2904 & 0.0341 \\ 0.5633 & 0.0511 & 0.3086 & 0.0362 \\ 0.5633 & 0.0511 & 0.3086 & 0.0362 \\ 0.5374 & 0.0488 & 0.2944 & 0.0346 \end{bmatrix}$$

$$(U - Q) = \begin{bmatrix} 0.4700 & -0.0481 & -0.2904 & -0.0341 \\ -0.5633 & 0.9489 & -0.3086 & -0.0362 \\ -0.5633 & -0.0511 & 0.6914 & -0.0362 \\ -0.5374 & -0.0488 & -0.2944 & 0.9654 \end{bmatrix}$$

Using MATLAB computation, we deduce the inverse of the matrix (U - Q) which is shown below:

$$F = (U - Q)^{-1} = \begin{bmatrix} 0.4700 & -0.0481 & -0.2904 & -0.0341 \\ -0.5633 & 0.9489 & -0.3086 & -0.0362 \\ -0.5633 & -0.0511 & 0.6914 & -0.0362 \\ -0.5374 & -0.0488 & -0.2944 & 0.9654 \end{bmatrix}^{-1}$$

$$F = (U - Q)^{-1} = \begin{bmatrix} 8.0018 & 0.6354 & 3.8362 & 0.4503 \\ 7.4409 & 1.6752 & 4.0768 & 0.4785 \\ 7.4409 & 0.6752 & 5.0768 & 0.4785 \\ 7.0995 & 0.6443 & 3.8897 & 1.4566 \end{bmatrix}$$

APPENDIX C Computation of the Resolution Matrix G

$$R = \begin{bmatrix} 0.0454 & 0.0384 & 0.0137 \\ 0.0000 & 0.0408 & 0.0000 \\ 0.0000 & 0.0408 & 0.0000 \\ 0.0460 & 0.0389 & 0.0000 \end{bmatrix}$$

$$G = \begin{bmatrix} 8.0018 & 0.6354 & 3.8362 & 0.4503 \\ 7.4409 & 1.6752 & 4.0768 & 0.4785 \\ 7.4409 & 0.6752 & 5.0768 & 0.4785 \\ 7.0995 & 0.6443 & 3.8897 & 1.4566 \end{bmatrix} * \begin{bmatrix} 0.0454 & 0.0384 & 0.0137 \\ 0.0000 & 0.0408 & 0.0000 \\ 0.0000 & 0.0408 & 0.0000 \\ 0.0460 & 0.0389 & 0.0000 \end{bmatrix}$$

Computations with MATLAB, give the solution matrix G, as

$$G = \begin{bmatrix} 0.3840 & 0.5072 & 0.1096 \\ 0.3598 & 0.5390 & 0.1019 \\ 0.3598 & 0.5390 & 0.1019 \\ 0.3893 & 0.5143 & 0.0973 \end{bmatrix}$$

APPENDIX D Computation of Variances

$$F_{dg} = \begin{bmatrix} 8.0018 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.6752 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 5.0768 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.4566 \end{bmatrix}$$

$$2\ F_{dg} = \begin{bmatrix} 16.0036 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 3.3504 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 10.1536 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.9132 \end{bmatrix}$$ by multiplying F_{dg} by 2.

$$(2\ F_{dg} - U) = \begin{bmatrix} 16.0036 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 3.3504 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 10.1536 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.9132 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(2\ F_{dg} - U) = \begin{bmatrix} 15.0036 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 2.3504 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 9.1536 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.9132 \end{bmatrix}$$

Then, we find the square of the fundamental matrix F_{sq} given by

$$F_{sq} = F^2 = \begin{bmatrix} 8.0018 & 0.6354 & 3.8362 & 0.4503 \\ 7.4409 & 1.6752 & 4.0768 & 0.4785 \\ 7.4409 & 0.6752 & 5.0768 & 0.4785 \\ 7.0995 & 0.6443 & 3.8897 & 1.4566 \end{bmatrix}^2$$

which gives

$$F_{sq} = \begin{bmatrix} 64.0288 & 0.4037 & 14.7164 & 0.2028 \\ 55.3670 & 2.8063 & 16.6203 & 0.2290 \\ 55.3670 & 0.4559 & 25.7739 & 0.2290 \\ 50.4029 & 0.4151 & 15.1298 & 2.1217 \end{bmatrix}$$

The variance of the fundamental matrix given by $F_v = F\,(2F_{dg} - U) - F_{sq}$ is

$$= \begin{bmatrix} 8.0018 & 0.6354 & 3.8362 & 0.4503 \\ 7.4409 & 1.6752 & 4.0768 & 0.4785 \\ 7.4409 & 0.6752 & 5.0768 & 0.4785 \\ 7.0995 & 0.6443 & 3.8897 & 1.4566 \end{bmatrix} . (2 * \begin{bmatrix} 8.0018 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.6752 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 5.0768 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.4566 \end{bmatrix} -$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}) - \begin{bmatrix} 64.0288 & 0.4037 & 14.7164 & 0.2028 \\ 55.3670 & 2.8063 & 16.6203 & 0.2290 \\ 55.3670 & 0.4559 & 25.7739 & 0.2290 \\ 50.4029 & 0.4151 & 15.1298 & 2.1217 \end{bmatrix}$$

$$F_v = \begin{bmatrix} 56.0270 & 1.0897 & 20.3986 & 0.6587 \\ 56.2733 & 1.1311 & 20.6971 & 0.6865 \\ 56.2733 & 1.1311 & 20.6971 & 0.6865 \\ 56.1152 & 1.0992 & 20.4750 & 0.6651 \end{bmatrix}$$

In the same vein, we compute the variance of the transition among non-absorbing states. This component column vector μ_v, is given by $\mu_v = (2F - U)\,\mu - \mu^2$. The other component column vector μ, specifies the directional movements and is also given by

$\mu = F\,v$

where v is an eigenvector given by:

$$v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$ and we already know F, the fundamental matrix.

Therefore,

$$\mu = \begin{bmatrix} 8.0018 & 0.6354 & 3.8362 & 0.4503 \\ 7.4409 & 1.6752 & 4.0768 & 0.4785 \\ 7.4409 & 0.6752 & 5.0768 & 0.4785 \\ 7.0995 & 0.6443 & 3.8897 & 1.4566 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 12.9237 \\ 13.6714 \\ 13.6714 \\ 13.0901 \end{bmatrix}.$$

If we ignore the effect imposed by rounding off, then we see that

$$\mu = \begin{bmatrix} 13.0000 \\ 13.0000 \\ 13.0000 \\ 13.0000 \end{bmatrix}$$

The other component column vector μ**,** which specifies the directional movements resonates the popular eigenvalue problem scenario of the type Bz = λz where B = Fundamental matrix F, and *z* represents the transpose of an eigenvector and λ denotes the eigenvalue. Mathematically,

B = F;

$z = [1,1,1,1]^T$

λ = eigenvalue.

This implies that

$Bz - \lambda I z = 0$

$(B - \lambda I)z = 0$ which implies that $F v = \lambda v$

This gives ***λ = 13*** and v is eigenvector from equation 18.

$$\mu^2 = \mu_{sq} = \begin{bmatrix} 12.9237 \\ 13.6714 \\ 13.6714 \\ 13.0901 \end{bmatrix}^2$$

$$\mu_{sq} = \begin{bmatrix} 167.0220 \\ 186.9072 \\ 186.9072 \\ 171.3507 \end{bmatrix}$$

Re-visiting the equation $\mu_v = (2F - U)\mu - \mu^2$, and using MATLAB,

$$(2F - U) = \begin{bmatrix} 15.0036 & 1.2708 & 7.6724 & 0.9006 \\ 14.8818 & 2.3504 & 8.1536 & 0.9570 \\ 14.8818 & 1.3504 & 9.1536 & 0.9570 \\ 14.1990 & 1.2886 & 7.7794 & 1.9132 \end{bmatrix}$$

$$(2F - U)\mu = \begin{bmatrix} 15.0036 & 1.2708 & 7.6724 & 0.9006 \\ 14.8818 & 2.3504 & 8.1536 & 0.9570 \\ 14.8818 & 1.3504 & 9.1536 & 0.9570 \\ 14.1990 & 1.2886 & 7.7794 & 1.9132 \end{bmatrix} \cdot \begin{bmatrix} 12.9237 \\ 13.6714 \\ 13.6714 \\ 13.0901 \end{bmatrix}$$

$$(2F - U)\mu = \begin{bmatrix} 327.9570 \\ 348.4595 \\ 348.4595 \\ 332.5199 \end{bmatrix}$$

$$\mu_v = \begin{bmatrix} 327.9570 \\ 348.4595 \\ 348.4595 \\ 332.5199 \end{bmatrix} - \begin{bmatrix} 167.0220 \\ 186.9072 \\ 186.9072 \\ 171.3507 \end{bmatrix} = \begin{bmatrix} 160.9350 \\ 161.5524 \\ 161.5524 \\ 161.1691 \end{bmatrix}$$

APPENDIX E Annual Breakdown of the Manpower data

The annual breakdown of the manpower raw data is embedded below as an attachment.

Manpower Raw Data.xlsx